Stability of the P to S energy ratio in the diffusive regime

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In the presence of inhomogeneities the propagation of elastic wave energy can be modeled by radiative transport equations. This is a good approximation when (i) typical wavelengths are short compared to the overall propagation distance, (ii) correlation lengths are comparable to wavelengths so that the inhomogeneities have appreciable effect and (iii) the fluctuations are weak. The relevant transport equations are derived in [1] starting from the elastic wave equations in an unbounded medium.

There is one consequence of the transport equations that may be important in understanding from first principles the stability of the P/Lg ratio that has proven so useful in yield estimation [2]. This is the fact that over distances (and times) that are long compared to the transport mean free path (transport mean free time), which is the diffusive regime, the P to S energy conversion by the random inhomogeneities equilibrates in a universal way, **independent of the details of the scattering**. There is an equipartition of energy [1] that leads to the relation

$$E_P(t,\mathbf{x}) = \frac{v_S^3}{2v_P^3} E_S(t,\mathbf{x}). \tag{1}$$

Here E_P and E_S are the P and S spatial energy densities, and v_P and v_S are the P and S wave speeds, respectively. For typical values of the P and S speeds this relation becomes $E_S \sim 10E_P$, which is in general agreement with observations.

The above equipartition law was derived when P to S mode conversion is generated by volume scattering. To get the correct equipartition law when Lg waves are present we have to model correctly the elastic wave scattering process in the crustal region and get the relevant radiative transport equations. That is, we must account correctly for the free surface and for the crustal waveguide in the radiative transport approximation. We are studying this problem at present.

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Introduction

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$$\mathcal{E}_P(t,\mathbf{x}) = \frac{v_S^3}{2v_P^3}\mathcal{E}_S(t,\mathbf{x}). \tag{1}$$

Here $\mathcal{E}_P$ and $\mathcal{E}_S$ are the P and S spatial energy densities, and v_P and v_S are the P and S wave speeds, respectively. For typical values of the P and S speeds this relation becomes $\mathcal{E}_S \sim 10\mathcal{E}_P$, which is in general agreement with observations.

The above equipartition law was derived when P to S mode conversion is generated by volume scattering. To get the correct equipartition law when Lg waves are present we have to model correctly the elastic wave scattering process in the crustal region and get the relevant radiative transport equations. That is, we must account correctly for the free surface and for the crustal waveguide in the radiative transport approximation. We are studying this problem at present.

Objective: Radiative Transport for Elastic Waves

Radiative Transport Equations.

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The theory of radiative transport was originally developed to describe how light energy propagates through a turbulent atmosphere. It is based upon a linear transport equation for the angularly resolved energy density and was first derived phenomenologically at the beginning of this century [3,4]. We show in [1] how this theory can be derived from the governing equations for light and for other waves of any type, in a randomly inhomogeneous medium. Our results take into account nonuniformity of the background medium, scattering by random inhomogeneities, the effect of polarization, the coupling of different types of waves, etc. The main new application is to elastic waves, in which shear waves exhibit polarization effects while the compressional waves do not, and the two types of waves are coupled. We also analyze solutions of the transport equations at long times and long distances and show that they have diffusive behavior.

Transport equations arise because a wave with wave vector $\mathbf{k}'$ at a point $\mathbf{x}$ in a randomly inhomogeneous medium may be scattered into any direction $\hat{\mathbf{k}}$ with wave vector $\mathbf{k}$. Therefore one must consider the angularly resolved, wave vector dependent, scalar energy density $a(t,\mathbf{x},\mathbf{k})$ defined for all $\mathbf{k}$ at each point $\mathbf{x}$ and time t. Energy conservation is expressed by the transport equation

$$\begin{aligned}\frac{\partial a(t,\mathbf{x},\mathbf{k})}{\partial t} &+ \nabla_{\mathbf{k}}\omega(\mathbf{x},\mathbf{k})\cdot\nabla_{\mathbf{x}}a(t,\mathbf{x},\mathbf{k}) - \nabla_{\mathbf{x}}\omega(\mathbf{x},\mathbf{k})\cdot\nabla_{\mathbf{k}}a(t,\mathbf{x},\mathbf{k})\\ &= \int_{\mathbb{R}^3}\sigma(\mathbf{x},\mathbf{k},\mathbf{k}')a(t,\mathbf{x},\mathbf{k}')d\mathbf{k}' - \Sigma(\mathbf{x},\mathbf{k})a(t,\mathbf{x},\mathbf{k}).\end{aligned} \tag{2}$$

Here $\omega(\mathbf{x},\mathbf{k})$ is the frequency at $\mathbf{x}$ of the wave with wave vector $\mathbf{k}$, $\sigma(\mathbf{x},\mathbf{k},\mathbf{k}')$ is the differential scattering cross-section, the rate at which energy with wave vector $\mathbf{k}'$ is converted to wave energy with wave vector $\mathbf{k}$ at position $\mathbf{x}$, and

$$\int\sigma(\mathbf{x},\mathbf{k}',\mathbf{k})d\mathbf{k}' = \Sigma(\mathbf{x},\mathbf{k}) \tag{3}$$

is the total scattering cross-section. Both σ and Σ are nonnegative and σ is usually symmetric in $\mathbf{k}$ and $\mathbf{k}'$. For an acoustic wave the differential scattering cross-section is given by

$$\begin{aligned}\sigma(\mathbf{x},\mathbf{k},\mathbf{k}') = &\Big((\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')^2\hat{R}_{\rho\rho}(\mathbf{k}-\mathbf{k}') + 2(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')\hat{R}_{\rho\kappa}(\mathbf{k}-\mathbf{k}') + \hat{R}_{\kappa\kappa}(\mathbf{k}-\mathbf{k}')\Big)\\ &\cdot\frac{\pi v^2(\mathbf{x})|\mathbf{k}|^2}{2}\delta(v(\mathbf{x})|\mathbf{k}| - v(\mathbf{x})|\mathbf{k}'|),\end{aligned} \tag{4}$$

where $\hat{R}_{\rho\rho}$, $\hat{R}_{\rho\kappa}$ and $\hat{R}_{\kappa\kappa}$ are the power spectra of the fluctuations of the density ρ and compressibilty κ defined in [1]. The left side of (2) is the total time derivative of $a(t,\mathbf{x},\mathbf{k})$ at a point moving along a ray in phase space $(\mathbf{x},\mathbf{k})$, which means that the frequency of the ray is adjusting to the appropriate local value. The right side of (2) represents the effects of scattering.

The transport equation (2) is conservative because

$$\iint a(t,\mathbf{x},\mathbf{k})d\mathbf{x}d\mathbf{k} = \text{const}$$

when the total scattering cross-section is given by (3). For simplicity we will assume that we do not have intrinsic attenuation. However, attenuation is easily accounted for by letting the total scattering cross-section be the sum of two terms

$$\Sigma(\mathbf{x},\mathbf{k}) = \Sigma_{sc}(\mathbf{x},\mathbf{k}) + \Sigma_{ab}(\mathbf{x},\mathbf{k})$$

where $\Sigma_{sc}(\mathbf{x},\mathbf{k})$ is the total cross-section due to scattering and is given by (3) and $\Sigma_{ab}(\mathbf{x},\mathbf{k})$ is the attenuation rate.

The reason that power spectral densities of the inhomogeneities determine the scattering cross-section (4) is seen most easily from a Born expansion of the wave equations when the inhomogeneities are weak. This is because the single scattering approximation of (2) and the second moments of the single scattering approximation for the underlying wave equations (the Born expansion) must be the same. The latter are determined by the power spectra of the inhomogeneities. In the same manner we can explain the appearance of the delta function in the cross-section (4) when the random inhomogeneities do not depend on time and therefore the frequencies are unchanged by the scattering. The transport equation (2) arises also when the waves are scattered by discrete scatterers that are randomly distributed in the medium. In this case the scattering cross-section (4) is the same as the cross-section of a single scatterer times the density of scatterers. We will deal only with continuous random media in [1].

Equation (2) has been derived from equations governing the particular wave motion under consideration by various authors such as Stott [5], Watson [6,7,8,9], Barabanenkov et.al. [10], Besieris and Tappert [11], Howe [12], Ishimaru [13] and Kohler et. al. [14] with a recent survey presented in [15]. These derivations also determine the functions $\omega(\mathbf{x},\mathbf{k})$ and $\sigma(\mathbf{x},\mathbf{k},\mathbf{k})$ and show how a is related to the wave field. In [1] we derive (2) and these functions as a special case of a more general theory. We expect that radiative transport equations will provide a good description of wave energy transport when, as mentioned in the Introduction, (i) typical wavelengths are short compared to macroscopic features of the medium (high frequency approximation), (ii) correlation lengths of the inhomogeneities are comparable to wavelengths and (iii) the fluctuations of the inhomogeneities are weak. It is difficult to compare wavelengths with correlation lengths in general because both can vary over very broad and overlapping ranges. Condition (ii) is important because it allows overlapping and therefore strong interaction between the waves and the inhomogeneities, which is the most interesting and difficult case to analyze. In addition to these three conditions, the inhomogeneities must not be too anisotropic because it is well known that in layered random media, for example, we have wave localization even with weak fluctuations, which is quite differnt from wave transport phenomena [16]. When the fluctuations are strong we can have wave localization even when the inhomogeneities are isotropic [17,18].

We also analyze the diffusive behavior of solutions of (2) which emerges at times and distances that are long compared to a typical transport mean free time $1/\Sigma$ and a typical transport mean free path $|\nabla_{\mathbf{k}}\omega|/\Sigma$, respectively. In this regime the phase space energy density $a(t,\mathbf{x},\mathbf{k})$ is approximately independent of the direction of the wave vector $\mathbf{k}$, $a(t,\mathbf{x},\mathbf{k}) \sim \bar{a}(t,\mathbf{x},|\mathbf{k}|)$ and in the simplest, spatially homogeneous case $\bar{a}$ satisfies the diffusion equation

$$\frac{\partial \bar{a}}{\partial t} = \nabla_{\mathbf{x}} \cdot (D \nabla_{\mathbf{x}} \bar{a}) \tag{5}$$

with a constant diffusion coefficient $D = D(|\mathbf{k}|)$ that is determined by the differential scattering cross-section σ and is given in [1]. Diffusion approximatons for scalar transport equations are well known [19], including their behavior near boundaries [20,21]. We show that diffusion approximations are also valid for the more general transport equations that arise for electromagnetic and elastic waves.

Transport Theory for Elastic Waves.

Radiative transport theory was first used in seismology by R.S. Wu [22]. The stationary, scalar transport equation was used to successfully assess scattering and intrinsic attenuation (the albedo) in several papers [23-28] and the time dependant scalar transport equation was used by Zeng, Su and Aki [29], Zeng [30] and Hoshiba [31]. In all these papers the vector nature of the underlying elastic wave motion was not taken into consideration. Mode conversion for surface waves was considered in a phenomenological way by Chen and Aki in [32] and general mode conversion between longitudinal compressional or P waves and transverse shear or S waves was considered by Sato in [33] and by Zeng in [34]. However, the transport equations proposed phenomenologically in [33,34] do not account for polarization of the shear waves. Starting from the elastic wave equations in a random medium we derive in [1] a system of transport equations that accounts correctly for P to S mode conversion and for polarization effects.

Longitudinal P waves propagate with local speed $v_P(\mathbf{x}) = \sqrt{(2\mu(\mathbf{x}) + \lambda(\mathbf{x}))/\rho(\mathbf{x})}$ and transverse shear or S waves that can be polarized propagate with local speed $v_S(\mathbf{x}) = \sqrt{\mu(\mathbf{x})/\rho(\mathbf{x})}$. The corresponding dispersion relations are $\omega_P = v_P|\mathbf{k}|$ and $\omega_S = v_S|\mathbf{k}|$, respectively. The P and S wave modes interact in an inhomogeneous medium because a P wave with a wavenumber $|\mathbf{k}|$ when scattered can generate an S wave with wavenumber $|\mathbf{p}|$ with the same frequency that is, $v_P(\mathbf{x})|\mathbf{k}| = v_S(\mathbf{x})|\mathbf{p}|$, and vice versa. These scattering processes conserve energy and the transport equations for P and S waves energy densities must therefore be coupled. The transport equation for the P wave energy density should be a scalar equation similar to (2) with an additional term that accounts for S to P energy conversion. Similarly, the transport equation for the S wave coherence matrix should be like Chandrasekhar's equation [3] with an additional term that accounts for P to S energy conversion. We show in [1] that this is indeed the case and we determine explicitly the form of the scattering cross-sections in terms of the power spectral densities of the material inhomogeneities.

The coupled transport equations for the P wave energy density $a^P(t, \mathbf{x}, \mathbf{k})$ and the 2×2 coherence matrix $W^S(t, \mathbf{x}, \mathbf{k})$ for the S waves have the form

$$\begin{aligned} &\frac{\partial a^P}{\partial t} + \nabla_{\mathbf{k}}\omega^P \cdot \nabla_{\mathbf{x}} a^P - \nabla_{\mathbf{x}}\omega^P \cdot \nabla_{\mathbf{k}} a^P \\ &\qquad = \int \sigma^{PP}(\mathbf{k}, \mathbf{k}') a^P(\mathbf{k}') d\mathbf{k}' - \Sigma^{PP}(\mathbf{k}) a^P(\mathbf{k}) \\ &\qquad + \int \sigma^{PS}(\mathbf{k}, \mathbf{k}')[W^S(\mathbf{k}')] d\mathbf{k}' - \Sigma^{PS}(\mathbf{k}) a^P(\mathbf{k}) \end{aligned} \tag{6a}$$

and

$$\frac{\partial W^S}{\partial t} + \nabla_{\mathbf{k}}\omega^S \cdot \nabla_{\mathbf{x}} W^S - \nabla_{\mathbf{x}}\omega^S \cdot \nabla_{\mathbf{k}} W^S + WN - NW$$
$$= \int \sigma^{SS}(\mathbf{k},\mathbf{k}')[W^S(\mathbf{k}')]d\mathbf{k}' - \Sigma^{SS}(\mathbf{k})W^S(\mathbf{k}) \tag{6b}$$
$$+ \int \sigma^{SP}(\mathbf{k},\mathbf{k}')[a^P(\mathbf{k}')]d\mathbf{k}' - \Sigma^{SP}(\mathbf{k})W^S(\mathbf{k}).$$

The differential scattering cross-section $\sigma^{PP}(\mathbf{k},\mathbf{k}')$ for P to P scattering is similar to (4) for scattering of scalar waves and the differential scattering tensor $\sigma^{SS}(\mathbf{k},\mathbf{k}')$ is similar to Chandrasekhar's tensor [3]. They have the form

$$\sigma^{PP}(\mathbf{k},\mathbf{k}') = \sigma_{pp}(\mathbf{k},\mathbf{k}')\delta(v_P|\mathbf{k}| - v_P|\mathbf{k}'|) \tag{7}$$

and

$$\begin{aligned}\sigma^{SS}(\mathbf{k},\mathbf{k}')[W(\mathbf{k}')] = \{&\sigma_{ss}^{TT}T(\mathbf{k},\mathbf{k}')W(\mathbf{k}')T(\mathbf{k}',\mathbf{k}) + \sigma_{ss}^{\Gamma\Gamma}\Gamma(\mathbf{k},\mathbf{k}')W(\mathbf{k}')\Gamma(\mathbf{k}',\mathbf{k})\\ &+\sigma_{ss}^{\Gamma T}(T(\mathbf{k},\mathbf{k}')W(\mathbf{k}')\Gamma(\mathbf{k}',\mathbf{k}) + \Gamma(\mathbf{k},\mathbf{k}')W(\mathbf{k}')T(\mathbf{k}',\mathbf{k})\}\\ &\cdot\delta(v_S|\mathbf{k}| - v_S|\mathbf{k}'|),\end{aligned} \tag{8}$$

where the 2×2 matrices $T(\mathbf{k},\mathbf{k}')$ and $\Gamma(\mathbf{k},\mathbf{k}')$ are defined by

$$T_{ij}(\mathbf{k},\mathbf{k}') = \mathbf{z}^{(i)}(\mathbf{k})\cdot\mathbf{z}^{(j)}(\mathbf{k}') \tag{9}$$

and

$$\Gamma_{ij}(\mathbf{k},\mathbf{k}') = (\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')(\mathbf{z}^{(i)}(\mathbf{k})\cdot\mathbf{z}^{(j)}(\mathbf{k}')) + (\hat{\mathbf{k}}\cdot\mathbf{z}^{(j)}(\mathbf{k}'))(\hat{\mathbf{k}}'\cdot\mathbf{z}^{(i)}(\mathbf{k})) \tag{10}$$

with $(\hat{\mathbf{k}},\mathbf{z}^{(1)}(\mathbf{k}),\mathbf{z}^{(2)}(\mathbf{k}))$ the orthonormal propagation triple consisting of the direction of propagation $\hat{\mathbf{k}}$ and two transverse unit vectors $\mathbf{z}^{(1)}(\mathbf{k}),\mathbf{z}^{(2)}(\mathbf{k})$, which in polar coordinates are

$$\hat{\mathbf{k}} = \begin{pmatrix}\sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{pmatrix},\ \mathbf{z}^{(1)}(\mathbf{k}) = \begin{pmatrix}\cos\theta\cos\phi\\ \cos\theta\sin\phi\\ -\sin\theta\end{pmatrix},\ \mathbf{z}^{(2)}(\mathbf{k}) = \begin{pmatrix}-\sin\phi\\ \cos\phi\\ 0\end{pmatrix}. \tag{11}$$

The scalar functions σ_{pp} and σ_{ss} are given in terms of power spectral densities of the inhomogeneities in [1] The total scattering cross-sections Σ^{PP} and Σ^{SS} are the integrals of the corresponding differential scattering cross-sections, as in (3), since we assume that there is no intrinsic dissipation.

The coupling matrix N is given by

$$N(\mathbf{x},\mathbf{k}) = \sum_{i=1}^{3}\frac{\partial v(\mathbf{x})}{\partial x^i}|\mathbf{k}|\mathbf{z}^{(1)}(\mathbf{k})\cdot\frac{\partial\mathbf{z}^{(2)}(\mathbf{k})}{\partial k_i}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}. \tag{12}$$

The scattering cross-sections for the S to P and P to S coupling terms, σ^{PS} and σ^{SP}, respectively, have the form

$$\begin{aligned}\sigma^{PS}(\mathbf{k},\mathbf{k}')[W^S(\mathbf{k}')] &= \mathrm{Tr}(\sigma_{ps}(\mathbf{k},\mathbf{k}')\mathcal{G}(\mathbf{k},\mathbf{k}')[W^S(\mathbf{k}')])\delta(v_P|\mathbf{k}|-v_S|\mathbf{k}'|) \\ \sigma^{SP}(\mathbf{k},\mathbf{k}')[a^P(\mathbf{k}')] &= \sigma_{ps}(\mathbf{k},\mathbf{k}')\mathcal{G}(\mathbf{k}',\mathbf{k})[a^P(\mathbf{k}')I]\delta(v_S|\mathbf{k}|-v_P|\mathbf{k}'|)\end{aligned} \tag{13}$$

where the tensor $\mathcal{G}(\mathbf{k},\mathbf{k}')$ acts on 2×2 matrices

$$\mathcal{G}(\mathbf{k},\mathbf{k}')[X] = \frac{1}{2}(G(\mathbf{k},\mathbf{k}')X + XG(\mathbf{k},\mathbf{k}')) \tag{14}$$

with the 2×2 matrix G given by

$$G_{ij}(\mathbf{k},\mathbf{k}') = (\hat{\mathbf{k}}\cdot\mathbf{z}^{(i)}(\mathbf{k}'))(\hat{\mathbf{k}}\cdot\mathbf{z}^{(j)}(\mathbf{k}')). \tag{15}$$

The scalar function σ_{ps} is given explictly in terms of power spectral densities of the inhomogeneities in [1].

The geometrical meaning of the 2×2 matrices T, Γ and G that appear in the differential scattering cross-sections (8) and (13) is similar to the one for T that appears in Chandrasekhar's equations [3]. They arise from a single scattering event of P and S waves with wave vector $\mathbf{k}'$ that scatter to P and S waves with wave vector $\mathbf{k}$ and from the fact that the transport equations deal with quadratic field quanitties.

As for the scalar transport equation (3) and Chandrasekhar's equations [3], the elastic transport equations (6) simplify considerably in the regime where the diffusion approximation is valid that is, when the transport mean free path is small compared to the propagation distance. In this regime the P wave energy density $a^P(t,\mathbf{x},\mathbf{k})$ and the S wave coherence matrix $W^S(t,\mathbf{x},\mathbf{k})$ are independent of the direction of the wave vector $\mathbf{k}$, W^S is proportinal to the identity matrix

$$a^P(t,\mathbf{x},\mathbf{k}) \sim \phi(t,\mathbf{x},|\mathbf{k}|),\ W^S(t,\mathbf{x},\mathbf{k}) \sim w(t,\mathbf{x},|\mathbf{k}|)I \tag{16}$$

and in addition we have the relation

$$\phi(t,\mathbf{x},|\mathbf{k}|) = w(t,\mathbf{x},\frac{v_P|\mathbf{k}|}{v_S}). \tag{17}$$

with ϕ satisfying the diffusion equation (5). The diffusion coefficient $D(|\mathbf{k}|)$ is given explicitly in [1].

The integrated over $\mathbf{k}$ form of the relation (17) is

$$\mathcal{E}_P(t,\mathbf{x}) = \frac{v_S^3}{2v_P^3}\mathcal{E}_S(t,\mathbf{x}) \tag{18}$$

where $\mathcal{E}_P$ and $\mathcal{E}_S$ are the P and S wave spatial energy densities that are related to a^P and W^S by

$$\mathcal{E}_P(t,\mathbf{x}) = \int a^P(t,\mathbf{x},\mathbf{k})d\mathbf{k}$$

and

$$\mathcal{E}_S(t,\mathbf{x}) = \int \mathrm{Tr}W^S(t,\mathbf{x},\mathbf{k})d\mathbf{k},$$

respectively. From the point of view of seismological applications of transport theory, relation (18) is important because it predicts universal behavior of the P to S wave energy ratio, in the diffusive regime. When we use the typical S to P wave speed ratio of 1 to 1.7, relation (18) predicts $\mathcal{E}_S/\mathcal{E}_P \sim 10$. This is in general agreement with seismological data and it would be interesting to identify cases where $\mathcal{E}_S/\mathcal{E}_P$ stabilizes. This stabilization, which is derived here from first principles, is reminiscent of the important empirical observation of Hansen, Ringdal and Richards [2] regarding the stabilization of the Lg wave energy.

Research Accomplished: Transport in Unbounded Media, Equipartition

The main accomplishments of the research reported in [1] and summarized here are (i) the derivation from first principles of the correct radiative transport equations for elastic wave motion in unbounded media (2) the demonstration that polarization of shear waves is important and must be taken into consideration and (iii) the demonstration that in the diffusive regime there is a universal P to S wave energy stabilization. This energy equipartition phenomenon, although inuitively clear was not known before and the precise form (1) that it takes is not easy to guess. It is perhaps the simplest instance of many different energy equipartition laws that are valid in other complex situations, such as the ones encountered in crustal wave propagation, that have not been discovered yet.

Relevance: P to S Conversion Stabilization

The relevance to yield estimation is immediate because, as noted in [2] and elsewhere since that paper appeared, P to Lg energy stabilization is the basis for a very successful yield estimation method. We have shown P to S energy stabilization so far, but we believe that the mechanism that controls it, equilibration of multiple scattering effects, is the same for P to Lg stabilization although the precise energy relation analogous to (1) is not known and is likely to be more complicated. The authors of [2] understood qualitatively the role of multiple scattering in the stabilization of P to Lg energy but did not note its universality and independence from the detailed mechanism of the scattering.

Conclusions and Recommendations

The use of transport theory in seismology is a promising research frontier and some very basic issues in yield estimation and other applications have their roots in it and will benefit enormously from a deeper understanding of its implications. An important next step for us is the explicit treatment of P to Lg energy stabilization by modeling the crustal structure in a suitable way and by deriving the transport equations for wave propagation it. Our goal is to get the form of relations like (1) that are valid in this context for the various elastic wave modes.

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